
Adaptive Context Tree Weighting

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Abstract

We describe an adaptive context tree weighting (ACTW) algorithm, as an extension to the standard context tree weighting (CTW) algorithm. Unlike the standard CTW algorithm, which weights all observations equally regardless of the depth, ACTW gives increasing weight to more recent observations, aiming to improve performance in cases where the input sequence is from a non-stationary distribution. Data compression results show ACTW variants improving over CTW on merged files from standard compression benchmark tests while never being significantly worse on any individual file.¹

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Keywords

Data compression; universal code; prediction; Context Tree Weighting (CTW) algorithm.

¹The results in this article first appeared in the honour's thesis of the first author [O'N10].

1 Introduction

Data compression is the task of encoding a data source into a more compact representation. In this paper, we are mainly interested in the task of lossless data compression, which requires that the original data must be exactly reproducible from the compressed encoding. There are a number of different techniques for lossless data compression. Some of the more popular methods employed include Burrows-Wheeler transform encoders [BW94], those based on Lempel-Ziv coding [ZL77, ZL78], using Dynamic Markov compression (DMC) [CH87] or using prediction by partial matching (PPM) [CIW84]. Many data compressors make use of a concept called arithmetic coding [Ris76, RL79, WNC87], which when provided with a probability distribution for the next symbol can be used for lossless compression of the data. In general, however, the true distribution for the next symbol is unknown and must be estimated. For stationary distributions it is easy to estimate the true distribution, which makes arithmetic coding asymptotically optimal. For non-stationary distributions this is no longer the case, and it is this problem we tackle in this paper.

CTW is an online binary prediction algorithm first presented in [WST95]. The general idea is to assume bits are generated by an unknown prediction suffix tree [RST96] of bounded depth and then use a Bayesian mixture to perform prediction. The CTW algorithm does this by means of a weighted context tree, which efficiently represents the Bayesian mixture over all suffix trees of bounded depth. This allows CTW to perform updates in time that grows linearly with depth, whereas a naive approach would lead to double exponential time. As such, it is an efficient, general purpose sequence prediction method that has been shown to perform well both theoretically and in practice [BEYY04].

It has been shown that CTW can be applied to lossless data compression [WST97]. The standard CTW algorithm, however, was designed specifically for coding sequences from a stationary source, so it is not surprising that it may perform poorly when the sequence is generated by a non-stationary (e.g. drifting) source. In this paper, we address this problem by introducing the adaptive CTW algorithm, which quickly adapts to the current distribution by increasing the weight of more recent symbols. It has been shown in [O’N10] that this adaptive CTW algorithm can improve results when integrated with a Monte Carlo AIXI agent [VNH⁺11].

Structure of this paper. This paper begins with a brief description of the standard CTW algorithm (Section 2). We then introduce several variants of ACTW algorithm (Section 3). Experimental results as well as an analysis of the performance of the ACTW variants are given in Section 4. Conclusions are drawn in the final section.

2 The CTW algorithm

The CTW algorithm [WST95, WST97] is a theoretically well-motivated and efficient online binary sequence prediction algorithm. It uses Bayesian model averaging that computes a mixture over all prediction suffix trees [RST96] up to a certain depth, with greater prior weight given to simpler models.

Krichevsky-Trofimov estimator. The KT estimator [KT81] is obtained using a Bayesian approach by assuming a $(\frac{1}{2}, \frac{1}{2})$ -Beta prior on the parameter of a Bernoulli distribution. Let $y_{1:t}$ be a binary string containing a zeros and b ones. We write $P_{kt}(a, b)$ to denote $P_{kt}(y_{1:t})$. The KT estimator can be incrementally calculated by: $P_{kt}(a+1, b) = \frac{a+1/2}{a+b+1} P_{kt}(a, b)$ and $P_{kt}(a, b+1) = \frac{b+1/2}{a+b+1} P_{kt}(a, b)$ with $P_{kt}(0, 0) = 1$.

Context Tree Weighing algorithm. A context tree of depth D is a perfect binary tree of depth D , with the left edges labelled 1 and the right edges labelled 0. Let n be a node in the context tree and suppose $y_{1:t}$ is the sequence that has been seen so far and $[y_{1:t}]_n$ is the sequence of bits in $y_{1:t}$ that end up in n . The counts a_n and b_n , corresponding to the number of zeros and ones in $[y_{1:t}]_n$, are stored at each node n , and are updated as bits in the input sequence are observed. The KT estimate is calculated at each node n based on the attached counts a_n and b_n . Additionally, we introduce a weighted probability for each node n , which can be recursively calculated by

$$P_w^n(y_{1:t}) = \begin{cases} P_{kt}([y_{1:t}]_n) & \text{if } n \text{ is a leaf node} \\ \frac{1}{2} P_{kt}([y_{1:t}]_n) + \frac{P_w^{n_l}([y_{1:t}]_{n_l}) P_w^{n_r}([y_{1:t}]_{n_r})}{2} & \text{otherwise} \end{cases}$$

where n_l and n_r are left and right children of n respectively. The joint probability for the input sequence is then given by the weighted probability at the root node.

3 Adaptive CTW

Limitations of standard CTW. The CTW algorithm is limited by its use of KT estimators to approximate the current distribution. This is appropriate if the true distribution is stationary, but not in the non-stationary case. The problem is that the KT estimator is very slow to update once many samples have been collected, so it cannot quickly learn a change in distribution.

KT with a moving window and discounted KT. Adaptive schemes such as [Wil96] and [SM99] have been studied. They are more computationally expensive than the standard KT estimator and we want an adaptive scheme that comes at no extra cost in computation time or memory. First we motivate our method by considering the KT estimator with a moving window. We estimate the probability of the next bit using a standard KT estimator, but instead of using all the previous history, we only use the last k bits. Suppose a sequence is generated by $Bern(\theta)$,

with some $0 \leq \theta \leq 1$. There is no nice redundancy bound for all cases, as one may encounter strings like $0^k 1^k 0^k \dots$. We instead studied the expected redundancy for one bit, which is

$$R(k; \theta) = \sum_{a=0}^k \binom{k}{a} \theta^a (1-\theta)^{k-a} \left[\theta \log \frac{k+1}{k-a+\frac{1}{2}} + (1-\theta) \log \frac{k+1}{a+\frac{1}{2}} \right] - H[\theta],$$

where $H[\theta]$ is the entropy. This quantity can be upper bounded by $O(1/k)$, and a lower bound of the same order is given in [Kri98, Thm. 1]. A shortcoming of this method is that one has to keep a history of length k . We therefore consider a discounted version of the KT estimator and apply it to the standard CTW algorithm. The discounted KT estimator solves the problem of storing a history of length k while retaining the desired fixed effective horizon of the windowing method described above in the sense that only recent bits significantly affect the prediction. Let $\gamma \in [0,1)$ denote the discount rate and $y_{1:t}$ be a binary string. We store discounted count a_t and b_t , corresponding to the discounted number of zeros and ones in $y_{1:t}$. After we observe the next symbol, as with in the standard KT estimator, one of the counts a_t and b_t is incremented according to the observed symbol and KT estimate is calculated based on a_t and b_t . We then use the discount rate to update the counts a_t and b_t by

$$a_{t+1} := (1 - \gamma) a_t \quad b_{t+1} := (1 - \gamma) b_t$$

ACTW. The adaptive CTW algorithm differs from the standard CTW algorithm only in the use of the discounted KT estimator. Therefore the adaptation comes at no extra computation or memory cost over the standard algorithm.

The value of γ need not be a constant. In the following we consider a number of possibilities for assigning γ .

Fixed rate. The most basic of the adaptive methods implemented is the fixed rate adaptive method. In this method, the value of γ is fixed, and every update makes use of this same constant. Consider an observed sequence $y_{1:t}$, where $[y_{1:t}]_n$ denotes the sequence of bits in $y_{1:t}$ which end up in n . Let k be the length of $[y_{1:t}]_n$ and use $[y_{1:t}]_{n,i}$ to denote the i^{th} bit in this sequence. Then for a constant γ we get a weighting for the i^{th} bit in $[y_{1:t}]_n$ given by:

$$w_{n,i} = (1 - \gamma)^{k-i}$$

Clearly as k increases the weighting decreases. In the case of $\gamma = 0$ this reduces to the standard CTW algorithm. For $\gamma > 0$ we have a desired fixed effective horizon whose length is determined by γ . While this method is simple, it suffers from the necessity of choosing parameter γ , which determines the fixed effective horizon.

Sequence length based. For this method the discount rate becomes a function of the length of the sequence observed so far. If $y_{1:t}$ is the sequence observed so far, then updates occurring due to the observation of y_{t+1} use an discount rate given by

$$\gamma_{t+1} = ct^{-\alpha} \quad c, \alpha \in [0, 1)$$

If $\alpha=0$ then this reduces to a fixed-rate adaptive CTW with $\gamma=c$. For $\alpha>0$, the adaptive multiplier decreases over time. Therefore this method leads to an increasing effective horizon, though at the cost of decreasing the benefits of adaptivity after observing a long input sequence. If there is a long sequence of observations between a node n being updated, this method can also lead to significant variation between weightings assigned to observations at the node, even if no other observations for that context have been observed in the meantime.

Context visit-based. To overcome this variation in weightings when dealing with contexts for which few observations have been made we use a method where the discount rate becomes a function of how many times a context has been observed. Below we present three variations on this idea, which share the same overarching philosophy but use somewhat different approaches.

Partial-context visit-based. The first approach is the most obvious application of this principle. At a node n where $[y_{1:t}]_n$ has length k (i.e. k different bits have been observed that end up in n), we use an discount rate given by

$$\gamma_n = ck^{-\alpha} \quad c, \alpha \in [0, 1)$$

This is perhaps the most elegant approach to the situation described above. However we note that for nodes higher in the context tree, which have the greatest impact, the number of observations of these contexts will increase rapidly, and so the adaptive multiplier for these nodes will decrease faster. While this is not necessarily a problem we nevertheless investigate methods that do not have this property.

Full-context visit-based. In this method we calculate an discount rate at the leaf node corresponding to the current context as per the previous method. However, instead of repeating this process at each node on the path towards the root, we instead propagate the discount rate up the tree. In this way all nodes on the context path use the same discount rate calculated at the leaf node. As the discount rate is based on observations of the leaf node, the value used for nodes higher in the tree will not be so closely linked to the sequence length. Formally, when updating a leaf node n where $[y_{1:t}]_n$ has length k_n we use the discount rate

$$\gamma_n = ck_n^{-\alpha}$$

and for each node n' on the path from n to the root node λ we use

$$\gamma_{n'} = ck_n^{-\alpha}$$

Therefore the same discount rate is used for all nodes in the path. Thus for a weighted context tree of depth D the discount rate becomes a function of the number of observations of the current length D context.

Leaf-context visit-based. This method is similar to the full-context visit-based adaptive method, but uses an additive rather than multiplicative approach to updating the counts a and b . For the leaf node corresponding to the current context

the same discount rate is used as for the previous two approaches, but for nodes on the path towards the root we instead update the counts as the sum of the counts of its child nodes. More formally, when updating a leaf node n where $[y_{1:t}]_n$ has length k_n we use the discount rate

$$\gamma_n = ck_n^{-\alpha}$$

and for nodes n' on the path from n to the root node we update the counts $a_{n'}$ and $b_{n'}$ using

$$a_{n'} = a_{n'_l} + a_{n'_r} \quad b_{n'} = b_{n'_l} + b_{n'_r}$$

where n'_l and n'_r are the left and right children of n' respectively. In this way discount rate has no effect on the KT-estimator count contributions of any other depth D context. This approach also preserves the property of CTW where the counts a_n and b_n of a node n is equal to the sum of the counts for its child nodes.

4 Experiments

Test datasets. In this section, we evaluate the variants of ACTW against the standard CTW algorithm² across a range of test sets, including the following standard benchmarks: large calgary corpus [BWC89], canterbury corpus³ and single file compression (SFC) testset⁴. We also tested our algorithm on an assortment of different file types that were collected for testing compression performance with changing sources. Details are given in Table 1. The division of files is given in Table 2, with the sets concatenated in the order listed.

Comparison data compressors. For comparison purposes, we have chosen data compressors to cover a range of the most commonly used compression techniques, including LZW, gzip, LZMA, bzip2, PAQ8L.

ACTW variants. We abbreviate different adaptive CTW variants used in experiment as below:

- ACTW1 - *fixed rate adaptive CTW*, $\gamma=0.01$
- ACTW2 - *partial context visit based adaptive CTW*, $c=0.1$, $\alpha=0.33$
- ACTW3 - *partial context visit based adaptive CTW*, $c=0.1$, $\alpha=0.5$
- ACTW4 - *full context visit based adaptive CTW*, $c=0.1$, $\alpha=0.33$
- ACTW5 - *leaf context visit based adaptive CTW*, $c=0.1$, $\alpha=0.33$

The context tree depth for compression testing was set to 28 for all the different CTW and ACTW based compressors. The parameters are mildly tuned with the objective of not being significantly worse than CTW on any file. As a consequence we have more modest gains as well.

²We used a generic CTW implementation, not the highly turned one presented in [TVW97]

³<http://corpus.canterbury.ac.nz/>

⁴<http://www.maximumcompression.com/index.html>

Table 1: Assorted collection of test files

| Size | Name | Type |
|----------|-------------|-----------------------------------|
| 3639172 | book1.pdf | PDF file |
| 2685309 | book2.txt | ASCII text |
| 656896 | data1.xls | Microsoft Excel spreadsheet |
| 544768 | data2.xls | Microsoft Excel spreadsheet |
| 1841392 | exec1 | UNIX compiled executable |
| 2169915 | exec2.exe | exe executable |
| 6784000 | exec3.msi | msi executable |
| 718377 | flash.swf | Shockwave flash file |
| 345160 | foreign.hwf | foreign language file |
| 561000 | lib.dll | Microsoft Dynamic Link Library |
| 1601949 | pic1.png | PNG image |
| 1861255 | pic2.png | PNG image |
| 55832855 | pitches | pitch values of MIDI files |
| 635392 | pres.ppt | Microsoft PowerPoint presentation |
| 86948 | text.rtf | rich-text format text |
| 2440044 | vid1.avi | avi video file |
| 5167297 | vid2.mov | mov video file |

Table 2: Assorted test file merged sets (ordered)

| merge1 | merge2 | merge3 | merge4 |
|-----------|-----------|-------------|-----------|
| exec1 | pic1.png | foreign.hwp | data2.xls |
| vid1.avi | data1.xls | exec3.msi | vid2.mov |
| flash.swf | exec2.exe | pres.ppt | text.rtf |
| book1.pdf | book2.txt | pic2.png | lib.dll |

Experimental results⁵. From analysis of these results the partial context visit based adaptive CTW with parameters $c=0.1$, $\alpha=0.33$ (ACTW2) appeared to produce the best compression results. Note that we compare generic bitwise compressors ACTW and CTW with highly tuned ones. For example, bzip2 uses several layers of compression techniques stacked on top of each other during compression. This comparison is not entirely fair as CTW and ACTW are not tuned. Therefore the results should not be taken as an indication that ACTW and CTW are necessarily inferior compression techniques.

Table 3 shows the results for the selected compressors on the large Calgary corpus. We see that the adaptive modification leads to better compression in only four test files, while for the remaining fourteen, the standard CTW algorithm is slightly better. Nevertheless, the difference exceeds 1% (1.07%) only in the bib case. Despite the general trend favouring the standard CTW approach, when compressing the concatenation of the corpus, adaptive CTW gives an improvement of nearly 1.3%.

⁵The numbers in the tables are defined as $1 - \frac{\text{Compressed size}}{\text{Uncompressed size}}$. The larger the better.

Table 3: Compression result, Large Calgary corpus

| file | LZW | gzip | LZMA | bzip2 | PAQ8L | CTW | ACTW1 | ACTW2 | ACTW3 | ACTW4 | ACTW5 |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| bib | 58.18 | 68.49 | 72.55 | 75.31 | 81.27 | 69.18 | 68.81 | 68.11 | 68.67 | 66.75 | 68.66 |
| book1 | 58.75 | 59.24 | 66.04 | 69.74 | 74.92 | 67.53 | 67.09 | 66.94 | 67.36 | 66.07 | 67.29 |
| book2 | 58.86 | 66.16 | 72.21 | 74.23 | 80.05 | 68.55 | 68.40 | 68.27 | 68.53 | 67.51 | 68.59 |
| geo | 24.05 | 33.11 | 48.06 | 44.41 | 57.04 | 35.11 | 36.03 | 35.69 | 35.39 | 36.01 | 35.15 |
| news | 51.30 | 61.59 | 68.49 | 68.55 | 76.17 | 61.83 | 61.64 | 61.28 | 61.62 | 60.45 | 61.55 |
| obj1 | 34.67 | 51.99 | 56.33 | 49.84 | 65.22 | 41.85 | 42.23 | 42.20 | 42.12 | 42.34 | 41.87 |
| obj2 | 47.87 | 66.93 | 75.13 | 69.03 | 81.80 | 56.65 | 57.58 | 57.80 | 57.34 | 57.64 | 57.40 |
| paper1 | 52.83 | 65.06 | 67.58 | 68.85 | 75.38 | 63.16 | 62.93 | 62.23 | 62.68 | 60.89 | 62.65 |
| paper2 | 56.01 | 63.80 | 66.92 | 69.54 | 75.09 | 66.11 | 65.81 | 65.27 | 65.75 | 63.82 | 65.73 |
| paper3 | 52.36 | 61.10 | 63.40 | 65.96 | 71.99 | 61.91 | 61.61 | 60.92 | 61.41 | 59.32 | 61.45 |
| paper4 | 47.64 | 58.33 | 59.69 | 60.95 | 68.46 | 56.35 | 56.12 | 55.30 | 55.75 | 53.62 | 55.80 |
| paper5 | 44.96 | 58.21 | 59.47 | 59.54 | 67.39 | 52.97 | 52.83 | 52.30 | 52.61 | 51.02 | 52.47 |
| paper6 | 50.94 | 65.27 | 67.30 | 67.74 | 75.06 | 61.72 | 61.50 | 60.84 | 61.26 | 59.56 | 61.18 |
| pic | 87.88 | 89.00 | 92.04 | 90.30 | 95.61 | 90.06 | 90.11 | 90.33 | 90.29 | 90.22 | 90.31 |
| progc | 51.67 | 66.49 | 68.42 | 68.33 | 75.99 | 61.51 | 61.39 | 60.79 | 61.13 | 59.70 | 61.08 |
| progl | 62.11 | 77.29 | 79.18 | 78.26 | 85.18 | 71.58 | 71.47 | 71.15 | 71.46 | 70.10 | 71.53 |
| progp | 61.10 | 77.23 | 79.13 | 78.31 | 85.56 | 70.73 | 70.57 | 69.96 | 70.35 | 69.03 | 70.29 |
| trans | 59.19 | 79.74 | 82.25 | 80.90 | 87.58 | 70.76 | 70.61 | 69.94 | 70.35 | 68.89 | 70.32 |
| merge | 57.95 | 67.15 | 73.82 | 72.63 | 80.94 | 66.11 | 67.15 | 67.40 | 67.00 | 67.36 | 67.28 |

Table 4: Compression result, Canterbury corpus

| file | LZW | gzip | LZMA | bzip2 | paq8l | CTW | ACTW1 | ACTW2 | ACTW3 | ACTW4 | ACTW5 |
|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| alice29.txt | 59.07 | 64.21 | 68.13 | 71.59 | 76.96 | 69.23 | 68.85 | 68.42 | 68.91 | 67.15 | 68.93 |
| asyoulik.txt | 56.07 | 60.90 | 64.45 | 68.39 | 73.60 | 66.61 | 66.22 | 65.68 | 66.20 | 64.32 | 66.22 |
| cp.html | 54.00 | 67.49 | 69.14 | 69.01 | 76.07 | 62.72 | 62.43 | 61.48 | 62.06 | 60.03 | 61.95 |
| fields.c | 55.48 | 71.81 | 73.30 | 72.74 | 79.97 | 64.25 | 64.01 | 63.15 | 63.60 | 61.97 | 63.49 |
| grammar.lsp | 51.28 | 66.51 | 66.60 | 65.52 | 74.12 | 58.18 | 58.08 | 57.43 | 57.70 | 56.49 | 57.46 |
| kennedy.xls | 69.85 | 79.92 | 94.66 | 87.35 | 98.86 | 75.15 | 82.59 | 83.36 | 80.88 | 83.43 | 83.31 |
| lcet10.txt | 61.77 | 66.05 | 72.02 | 74.76 | 80.34 | 70.18 | 69.83 | 69.62 | 70.00 | 68.67 | 70.00 |
| plrabn12.txt | 59.12 | 59.49 | 65.68 | 69.79 | 74.69 | 68.37 | 67.88 | 67.74 | 68.19 | 66.65 | 68.13 |
| ptt5 | 87.88 | 89.00 | 92.04 | 90.30 | 95.61 | 90.06 | 90.11 | 90.33 | 90.29 | 90.22 | 90.31 |
| sum | 47.43 | 66.20 | 75.36 | 66.24 | 80.33 | 57.14 | 57.51 | 57.25 | 57.10 | 57.03 | 57.10 |
| xargs.l | 44.67 | 58.46 | 58.34 | 58.32 | 66.55 | 50.06 | 49.82 | 48.78 | 49.21 | 47.36 | 49.25 |

Table 5: Compression result, SFC testset

| file | LZW | gzip | LZMA | bzip2 | paq8l | CTW | ACTW1 | ACTW2 | ACTW3 | ACTW4 | ACTW5 |
|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| A10.jpg | 0.00 | 0.12 | -0.41 | 0.71 | 17.09 | -3.86 | -3.77 | -3.47 | -3.55 | -3.24 | -3.69 |
| AcroRd32.exe | 37.79 | 55.19 | 63.65 | 56.09 | 76.05 | 46.02 | 47.83 | 48.19 | 47.53 | 48.71 | 47.82 |
| english.dic | 62.69 | 74.18 | 79.06 | 69.96 | 90.47 | 65.18 | 75.87 | 76.67 | 74.43 | 76.14 | 76.74 |
| FlashMX.pdf | 0.00 | 15.26 | 18.11 | 15.82 | 21.27 | 8.62 | 8.92 | 9.30 | 9.17 | 9.58 | 9.07 |
| FP.LOG | 86.90 | 92.97 | 96.03 | 96.49 | 98.69 | 91.51 | 92.00 | 92.48 | 92.18 | 92.35 | 92.58 |
| MSO97.DLL | 22.74 | 42.07 | 51.95 | 44.19 | 65.28 | 36.65 | 37.70 | 37.85 | 37.40 | 38.17 | 37.58 |
| ohs.doc | 62.72 | 75.65 | 81.07 | 78.24 | 86.87 | 73.83 | 73.47 | 74.18 | 74.19 | 74.28 | 74.09 |
| rafale.bmp | 65.18 | 69.60 | 76.50 | 78.55 | 84.71 | 75.69 | 76.84 | 77.15 | 77.04 | 76.53 | 77.40 |
| vcfiu.hlp | 64.65 | 79.41 | 85.20 | 82.71 | 90.21 | 67.13 | 71.08 | 71.16 | 69.68 | 71.31 | 71.03 |
| world95.txt | 62.07 | 70.78 | 80.95 | 80.69 | 87.81 | 70.37 | 70.37 | 70.42 | 70.56 | 69.88 | 70.64 |
| merge | 55.01 | 71.56 | 76.68 | 74.53 | 82.77 | 64.30 | 67.86 | 68.39 | 67.30 | 68.73 | 68.14 |

Results for the Canterbury Corpus in Table 4 once again seems to generally favour the standard CTW algorithm, though again only by small amounts. One exception to this trend is the kennedy.xls file, where adaptive CTW modifications allow the space savings to be improved by over 8%. For two compressors based on

Table 6: Compression result, Gauntlet benchmark

| file | LZW | gzip | LZMA | bzip2 | paq8l | CTW | ACTW1 | ACTW2 | ACTW3 | ACTW4 | ACTW5 |
|---------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| abac | 99.45 | 99.88 | 99.94 | 99.98 | 99.95 | 99.98 | 98.73 | 99.59 | 99.91 | 99.57 | 99.63 |
| abba | 95.12 | 94.16 | 99.79 | 96.16 | 98.21 | 96.43 | 95.21 | 96.32 | 96.42 | 96.28 | 96.36 |
| book1x20 | 60.27 | 59.39 | 98.29 | 71.83 | 98.71 | 69.97 | 69.21 | 69.44 | 69.80 | 69.13 | 69.63 |
| fib_s14930352 | 99.22 | 99.27 | 99.85 | 99.99 | 99.96 | 95.42 | 94.12 | 95.31 | 95.41 | 95.28 | 95.35 |
| fss10 | 99.04 | 98.95 | 99.94 | 99.99 | 99.96 | 93.45 | 92.15 | 93.34 | 93.44 | 93.29 | 93.39 |
| fss9 | 98.51 | 98.93 | 99.94 | 99.98 | 99.96 | 93.45 | 92.15 | 93.27 | 93.43 | 93.18 | 93.36 |
| houston | 98.96 | 98.75 | 99.22 | 99.39 | 99.54 | 88.95 | 88.13 | 88.82 | 88.93 | 88.80 | 88.84 |
| paper5x80 | 77.28 | 98.75 | 99.48 | 98.02 | 99.57 | 82.79 | 81.85 | 81.45 | 82.27 | 80.78 | 81.92 |
| test1 | 95.98 | 99.54 | 99.96 | 99.86 | 99.96 | 91.94 | 90.96 | 91.28 | 91.76 | 91.11 | 91.37 |
| test2 | 95.98 | 99.54 | 99.96 | 99.86 | 99.96 | 91.96 | 91.01 | 91.35 | 91.80 | 91.18 | 91.45 |
| test3 | 0.00 | 3.93 | 98.06 | 92.37 | 99.96 | 75.98 | 75.27 | 73.56 | 74.70 | 72.61 | 74.47 |
| merge | 82.62 | 86.22 | 99.41 | 92.60 | 99.37 | 86.77 | 86.74 | 87.49 | 87.57 | 87.38 | 87.57 |

Table 7: Compression result, assorted files

| file | LZW | gzip | LZMA | bzip2 | paq8l | CTW | ACTW1 | ACTW2 | ACTW3 | ACTW4 | ACTW5 |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| book1.pdf | 0.00 | 22.82 | 25.06 | 23.11 | 27.37 | 15.73 | 16.71 | 17.16 | 16.84 | 17.47 | 16.89 |
| book2.txt | 60.75 | 62.82 | 71.39 | 73.41 | 79.61 | 69.99 | 69.65 | 69.80 | 70.04 | 69.22 | 70.05 |
| data1.xls | 65.04 | 71.02 | 81.73 | 76.45 | 89.67 | 68.20 | 70.98 | 71.18 | 70.04 | 71.51 | 70.88 |
| data2.xls | 64.85 | 73.26 | 85.17 | 76.96 | 93.53 | 66.95 | 69.64 | 69.90 | 68.64 | 70.24 | 69.66 |
| exec1 | 28.78 | 47.45 | 55.60 | 48.06 | 67.38 | 38.73 | 40.05 | 40.40 | 39.86 | 40.80 | 40.00 |
| exec2.exe | 0.00 | 1.01 | -0.07 | 0.22 | 1.52 | -9.19 | -9.05 | -8.55 | -8.70 | -8.15 | -8.87 |
| exec3.msi | 0.00 | 4.60 | 4.27 | 3.91 | 6.00 | -5.53 | -5.28 | -4.79 | -4.96 | -4.40 | -5.06 |
| flash.swf | 0.00 | 0.29 | -0.73 | -0.38 | 0.64 | -9.86 | -9.76 | -9.35 | -9.45 | -9.04 | -9.59 |
| foreign.hwp | 0 | 5.78 | 6.08 | 4.68 | 7.31 | -3.65 | -3.57 | -3.13 | -3.22 | -2.80 | -3.41 |
| lib.dll | 0.00 | 15.48 | 16.83 | 13.49 | 19.58 | 5.50 | 5.89 | 6.31 | 6.11 | 6.69 | 5.95 |
| pic1.png | 0.00 | 0.19 | -0.12 | -0.30 | 3.37 | -8.86 | -8.75 | -8.28 | -8.40 | -7.89 | -8.61 |
| pic2.png | 0.00 | 0.05 | 0.43 | 0.43 | 3.24 | -6.99 | -6.88 | -6.49 | -6.59 | -6.16 | -6.76 |
| pitches | 37.91 | 69.71 | 74.00 | 64.27 | 78.15 | 49.60 | 52.72 | 52.49 | 50.93 | 52.77 | 52.45 |
| pres.ppt | 0.00 | 61.72 | 66.14 | 62.12 | 74.81 | 49.86 | 50.13 | 50.00 | 50.01 | 49.94 | 49.78 |
| text.rtf | 67.39 | 84.55 | 85.81 | 85.69 | 89.70 | 77.48 | 77.24 | 76.70 | 77.24 | 75.64 | 76.99 |
| vid1.avi | 0.00 | 3.48 | 3.93 | 3.54 | 6.12 | -1.02 | -0.86 | -0.49 | -0.59 | -0.22 | -0.74 |
| vid2.mov | 0.00 | 1.76 | 3.11 | 2.12 | 7.91 | 0.42 | 0.60 | 0.87 | 0.77 | 1.32 | 0.62 |
| merge1 | 0.00 | 20.74 | 23.47 | 20.92 | 27.59 | 11.72 | 12.83 | 13.59 | 13.04 | 14.33 | 13.02 |
| merge2 | 5.00 | 30.08 | 33.94 | 33.42 | 38.92 | 26.01 | 26.40 | 26.80 | 26.63 | 27.10 | 26.47 |
| merge3 | 0 | 7.52 | 7.70 | 6.85 | 9.95 | -2.73 | -2.39 | -1.89 | -2.11 | -1.52 | -2.17 |
| merge4 | 0.00 | 10.20 | 12.46 | 10.27 | 17.34 | 6.21 | 7.01 | 7.43 | 6.98 | 8.13 | 6.96 |

the same principles this is a significant improvement.

The SFC test set results given in Table 5 show ACTW2 consistently outperforming standard CTW. Some of these are quite significant increases, including an improvement of over 2% for AcroRd32.exe, 4% for vcfu.hlp and 10.5% for english.dic.

Once again, considering the close relation to the standard CTW algorithm this is a very significant improvement in space savings. English.dic is an alphabetically sorted word file. It appears the adaptive modifications allow ACTW2 to better adapt to the changing distribution of the text in the file. This leads the ACTW2 to outperform bzip2 by around 5% while the standard CTW algorithm trailed bzip2 as much.

Again we see ACTW2 perform better when compressing the concatenated test set. However as ACTW2 was generally performing better on the individual files

this is not as indicative of how adaptation improves compression when using varied sources. One other observation of interest here is how ACTW2 is able to perform very closely to bzip2 for the rafale.bmp file. Something similar was seen in the Calgary corpus case, where ACTW2 gave better results than bzip2 for the pic file, also a bitmap image. Consulting the index it can be seen that this is not a special case of poor performance for the bzip2 compressor, but instead seems to indicate some property of the bitmap file format that makes it particularly suited to CTW based compression.

In the Gauntlet benchmark results seen in Table 6 we once more see CTW outperforming ACTW. While the margin between the two is quite small, CTW gives better compression for every individual file in this test set. However looking at the concatenated file results we see ACTW2 giving around a 0.8% improvement in space savings. As this cannot be the result of better compression for any individual file, it must indicate that the adaptive modifications allow ACTW2 to better respond to changing input distributions.

For the assorted file collection results given in Table 7 we see ACTW2 generally offering better compression than standard CTW, including for all four concatenated files tested here. Space saving increases of around 3% can be seen for both data1.xls and data2.xls. In combination with the improvement of more than 8% for kennedy.xls in the Canterbury corpus this suggests there might be something about the Microsoft Excel file format that makes it well suited to ACTW2.

5 Conclusion

We proposed the ACTW algorithm as an extension to the standard CTW algorithm that puts greater weighting on more recent observations. Four different versions of ACTW algorithms were tested to determine the effects on prediction performance. The performance of ACTW, especially partial context visit based adaptive CTW with $c = 0.1$, $\alpha = 0.33$, was promising with space saving improvements up to 10% when compared to the standard CTW algorithm. While the standard CTW was able to outperform ACTW for a number of files, the difference in space savings for these cases very rarely exceeded 1%. For concatenated files from varying sources, ACTW was seen to outperform standard CTW for all tests performed, demonstrating how the adaptive modifications to CTW allow better handling of changing source distributions. This improved performance for concatenated files was even observed when the standard CTW algorithm provided better compression for each of the individual files.

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